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3D numerical simulation of tsunami runup onto a complex beach

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ABSTRACT: Two 3D numerical models are applied to tsunami phenomena including runup onto a complex beach. Their governing equations are the continuity and Reynolds-averaged Navier-Stokes equations for incompressible fluids in porous media. In the first model water surface displacement is determined by the vertically integrated equation of continuity, while in the second by the 3D-VOF method. Seabed topography can be smoothly expressed with the porous model. These two models reproduce the existing hydraulic experiment, which treated the 1993 Hokkaido Nansei-Oki earthquake tsunami in Okushiri Island, where the extreme runup of about 32 m was discovered. The calculation results of water surface displacement generally show correspondence with the experimental data at off-peak times of water level when no breaking wave exists. The results calculated by these 3D models are also compared with those by the 2D long-wave model. The highest runup obtained by the VOF model indicates the full-scale value of about 30.6 m.

1 INTRODUCTION

When a submarine earthquake or a land slide occurs, the effects appear in various ways. Especially seawater threatens humankind near seashore as a tsunami. In coastal zones, the seawater flow shows complicated motion because of affectors to tsunamis, e.g. bottom configuration in both land and sea areas, river flow, astronomical tide, density stratification, etc. Around nearshore structures, fluid motion due to tsunamis deviates from simple flow, resulting in these structures under three-dimensionally distributed forces, whose mechanism should be known because it concerns weakness or inversion of the structures. We cannot evaluate, however, such a 3D distribution using the traditional nonlinear long-wave models. Moreover, from offshore to coastal zones, phenomena, which have various scales on space and time, should be solved efficiently and economically.

For these purposes a hybrid model which consists of 3D models, multilevel models and connection models has been developed by Kakinuma & Tomita (2005). The governing equations are the continuity and Reynolds-averaged Navier-Stokes equations for incompressible fluids with a porous model describing seabed topography smoothly. The numerical schemes are the finite difference methods with orthogonal grid systems.

The multilevel model is used for wide-area calculation as shown in Figure 1. We assume hydrostatic pressure in these areas, to which a fault model and a typhoon model are introduced. The connection model is applied to overlap regions for smooth connection between the multilevel model and the 3D model, where we solve pressure without assumption of hydrostatic pressure. By local application of this 3D model to narrower areas surrounded by multilevel areas, we can represent 3D characteristics of flow efficiently and economically around structures, over steep topography and so on.

For disaster prevention, many things are required to be stocked, e.g. food, facility, commodity, knowledge, information, attitude of mind, etc. This numerical calculation model is called as Storm surge and Tsunami simulator in Oceans and Coastal areas, STOC.

In this paper, only the 3D part of STOC was applied to simulate tsunami runup. In STOC we have two 3D models: when we calculate water surface distribution, we use 1) Vertically integrated equation of continuity or 2) 3D-VOF method. The first model is labeled as STOC-IC, while the second one as STOC-VF. We can choose between two. Calculation results were obtained to reproduce the tsunami runup measured in the large-scale tank (L 205 m, D 6 m and W 3.4m). This experiment was performed by Matsuyama & Tanaka (2001) to represent the extreme runup of about 32 m height discovered at the tip of a narrow gulley within a small cove near the village of Monai in Okushiri Island, Japan, due to the 1993 Hokkaido Nansei-Oki earthquake tsunami.

2 3D NUMERICAL SIMULATOR FOR FLUID MOTION

2.1 *Governing Equations*

In the numerical simulator, STOC, we solve the set of continuity and Reynolds-averaged Navier-Stokes equations, i.e.,

$$\frac{\partial \varepsilon}{\partial t} + \frac{\partial (\gamma_i u_i)}{\partial x_i} = 0, \tag{1}$$

$$\frac{\partial(\varepsilon u_i)}{\partial t} + \frac{\partial(\gamma_j u_i u_j)}{\partial x_j} + C_i = -\frac{\varepsilon}{\rho_0}\frac{\partial p}{\partial x_i} + \varepsilon\frac{\rho}{\rho_0} g_i + \frac{\partial}{\partial x_j}\left\{\gamma_j \nu_e \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\right\} \tag{2}$$

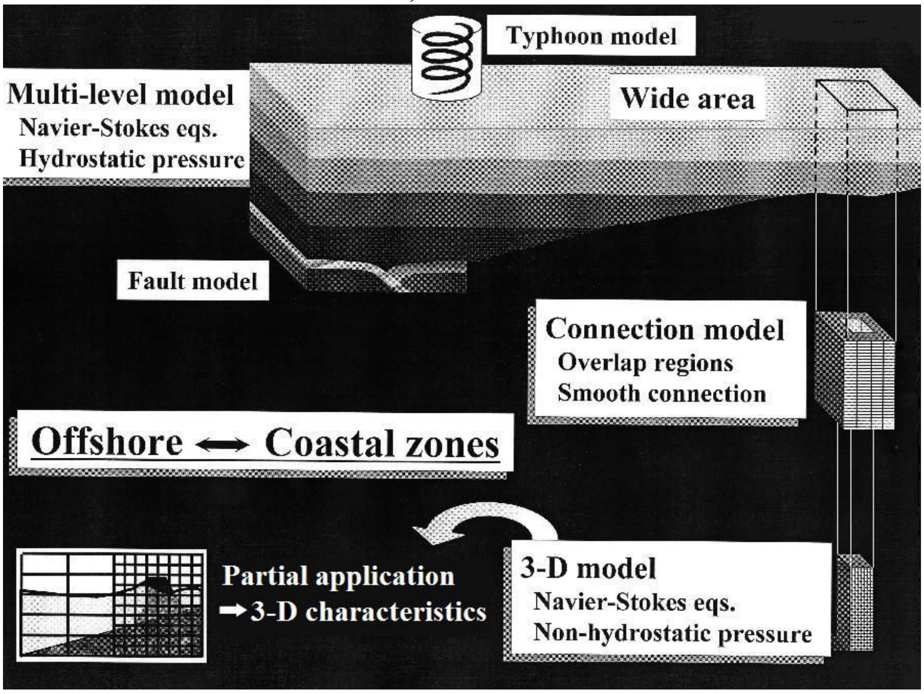


Figure 1. Diagram of STOC

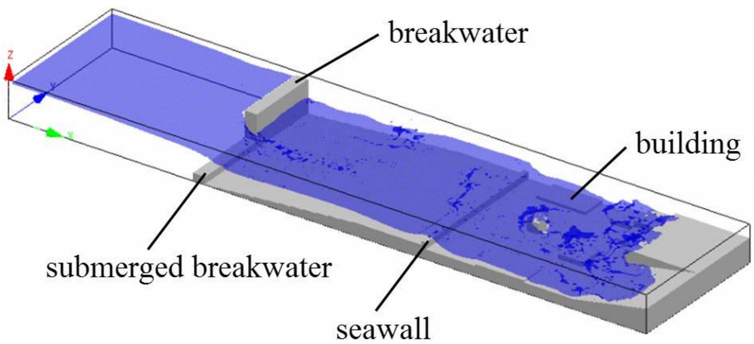


Figure 2. An example of calculation result by STOC-VF: Water surface of an imaginary tsunami

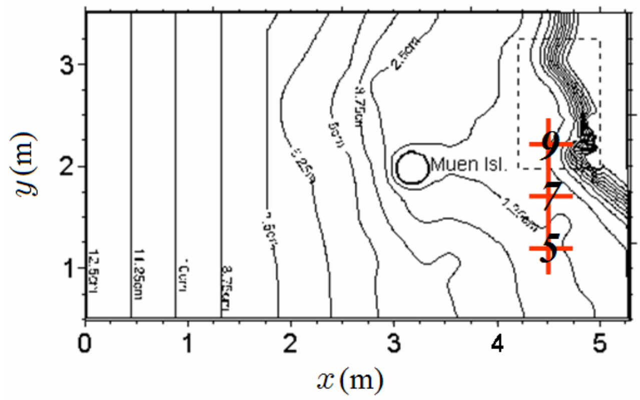
where x_i describes the Cartesian coordinate system, (x, y, z); u_i is velocity in the direction of x_i, (u, v, w); ρ is density; ρ_0 is a reference density; p is pressure; ε is porosity; γ_i is transmissivity in the direction of x_i; g_i is gravitational acceleration, $(0,0,-g)$; ν_e is total viscosity including dynamic viscosity and eddy viscosity; C_i is Coriolis term. A porous model is introduced to express the smooth shape of sea bottoms or structure faces.

We solve these governing equations using a finite difference method. In space, a staggered mesh is adopted, where for the diffusion terms the second-order central scheme is used, while for the advection terms a hybrid scheme, i.e., the first-order upwind scheme combined with the second-order central scheme, is utilized for stability. In time, a leapfrog method is used in STOC-IC, staggering calculation time of both water surface displacement and pressure and that of velocity by Δt, while SMAC method is used in STOC-VF.

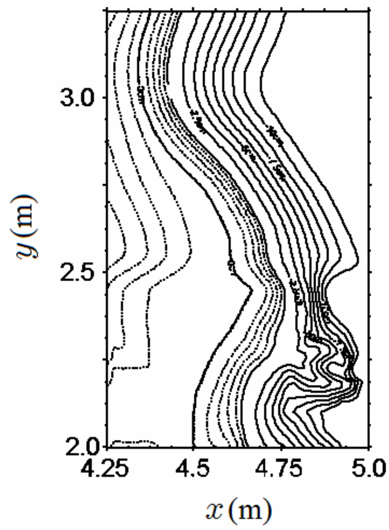
2.2 *Calculation Methods of Water Surface Displacement*

1) STOC-IC

After calculation in each cell at the current time-step, we should know the position of water surface at the next time-step. In STOC-IC we use the vertically Integrated equation of Continuity, i.e.,



(a) Bathymetry in the flume



(b) Detailed topography near the maximum-runup valley

Figure 3. Bathymetry and topography in the experiment

$$\gamma_z \frac{\partial \eta}{\partial t} + \frac{\partial}{\partial x}\int_{-h}^{\eta} \gamma_x u \, dz + \frac{\partial}{\partial y}\int_{-h}^{\eta} \gamma_y v \, dz = 0 \,, \qquad (3)$$

where η is water surface displacement; h is still water depth.

In runup regions, "Bucket-brigade method" is used; that is there exists water at the places where the water depth is larger than some reference value. Unfortunately this model is not applicable to cases where water surface elevation is described by a multivalued function of x_i, because Equation 3 is a vertically-integrated-type equation.

2) STOC-VF

On the other hand, STOC-VF is a 3D-VOF method (Arikawa et al., 2005), which is an extended version of the 2D-VOF method (CDIT, 2001), using a convection equation on F, i.e.,

Table 1. Wave gauge positions

Channel	x(m)	y(m)
1	4.521	0.196
2	4.521	0.446
3	4.521	0.696
4	4.521	0.946
5	4.521	1.196
6	4.521	1.446
7	4.521	1.696
8	4.521	1.946
9	4.521	2.196
10	4.521	2.446
11	4.521	2.969
12	4.521	2.946
13	4.521	3.196

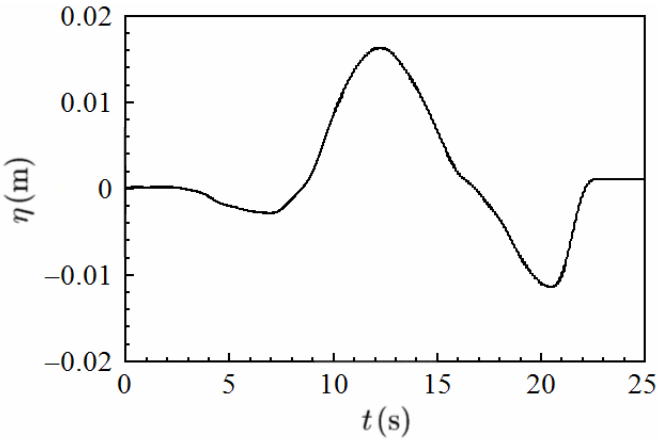


Figure 4. Incident wave condition

$$\frac{\partial(\varepsilon F)}{\partial t} + \frac{\partial(\gamma_i u_i F)}{\partial x_i} = \text{(source)}. \qquad (4)$$

Using this method, which works also when the water surface elevation is described by a multivalued function of x_i, we can represent breaking-wave phenomena of fluid motion as shown in Figure 2. This figure shows an imaginary tsunami attacking buildings in a town protected by a breakwater, a submerged breakwater and a seawall.

3 NUMERICAL SIMULATION OF TSUNAMI RUNUP ONTO A COMPLEX BEACH

3.1 *Calculation Condition*

We tried tsunami calculation using both STOC-IC and STOC-VF. The calculation problem is to reproduce the Monai runup in the 1/400 length-scale laboratory experiment by Matsuyama & Tanaka

Table 2. Calculation condition

Machine condition	
Computer	2.5 GHz desktop-type machine
Used memory	930～940 MB
Coding	FORTRAN 90 on LINUX
STOC-IC	
Δt	0.005～0.01 s (according to Courant number)
Δx & Δy	0.014 m
Δz	0.014 m for $z = -0.14 \sim 0.126$ m
Number of grid points	$393 \times 244 \times 20 = 1,917,840$
CPU time	17 hrs on slip bottom for $t = 0 \sim 25.0$ s 12 hrs on non-slip bottom for $t = 0 \sim 25.0$ s
STOC-VF	
Δt	0.001～0.005 s (according to Courant number)
Δx & Δy	0.014 m
Δz	0.017～0.008 m for $z = -0.15 \sim -0.024$ m 0.008 m for $z = -0.024 \sim 0.12$ m
Number of grid points	$393 \times 244 \times 28 = 2,684,976$
CPU time	46 hrs for $t = 0 \sim 24.0$ s

(2001), whose results were also compared with the calculation results by Yoneyama et al. (2002). Figure 3 shows the coastal topography used in the experiment. There are reflective and vertical sidewalls where y = 0 and 3.4 (m). The wave gauges of the experiment were set at the 13 points shown in Table 1, among which three gauges of Nos. ***5***, ***7*** and ***9*** are pointed in Figure 3(a).

The incident wave is given as shown in Figure 4 from offshore, where the water depth d = 13.5 cm. The calculation conditions are shown in Table 2.

Note that it is not easy to compare the CPU time for the calculation with STOC-IC and that with STOC-VF because Δt changes according to Courant number and the numbers of grid points are different.

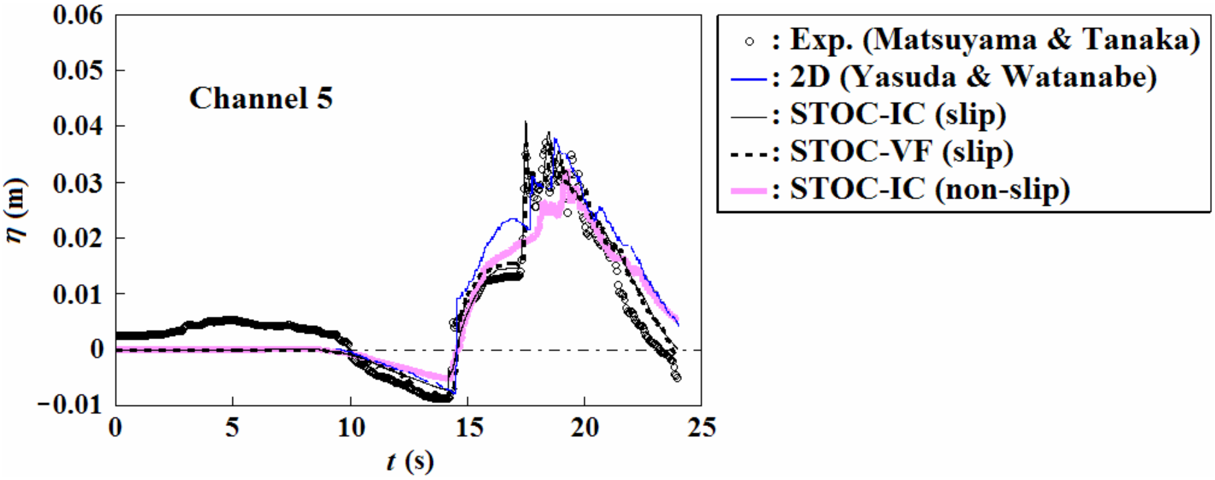
3.2 *Calculation Results*

1) Water level calculated by STOC-IC and STOC-VF under the slip-bed condition

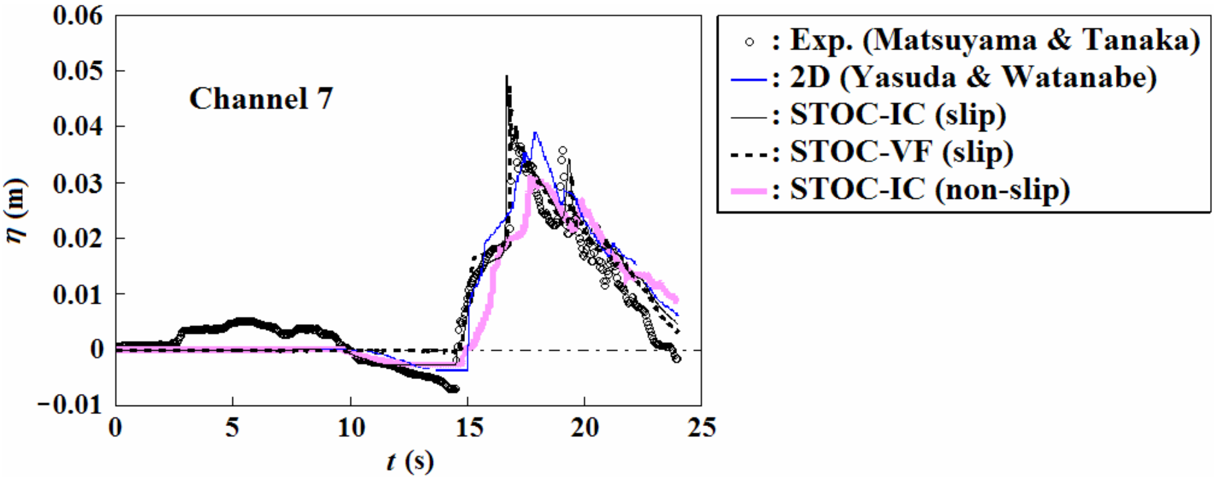
Figures 5(a), 5(b) and 5(c) show time variation of water level at Channels 5, 7 and 9, respectively. In these figures the calculation results by both STOC-IC and STOC-VF are compared with the experimental data. In STOC-VF, ν_e, which includes eddy viscosity, is equal to zero, while in STOC-IC an LES model is installed. Although there is such difference in treatment of turbulence between these two models, the results by STOC-VF and STOC-IC are similar except when t = 15 − 16.5 (s) at Channel 9, where STOC-IC cannot represent breaking phenomena without wave-breaking model.

2) Water level calculated by STOC-IC under the non-slip-bed condition

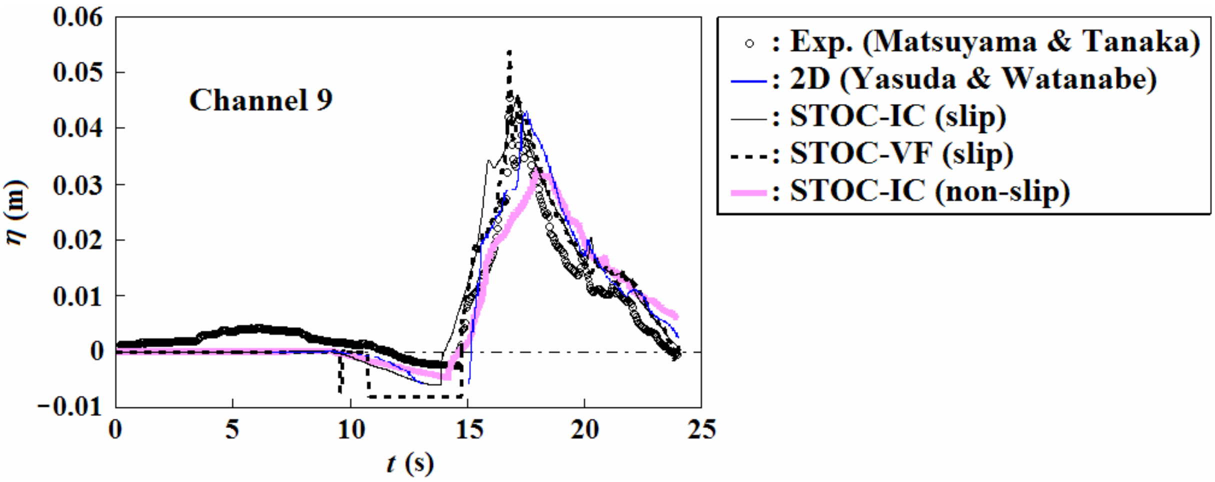
In Figure 5, in comparison with the experimental data, the calculation results of water level under the slip-bed condition show higher first-peak values, which appear as the first local maximal values in the experiment when $t \geqq 15.0$ s. Secondly we have tried STOC-IC under the non-slip-bed condition for the same incident waves. The time variation of water level on the non-slip bottom is shown in Figure 5 with the pink-colored lines. These calculation results get too lower water level than that observed in the experiment. This reason should be that we do not perform discretization of numerical meshes in the vertical direction inside the bottom boundary layer because it is efficient to use meshes whose vertical width is much larger than the vertically changing rate of velocity inside the boundary layer. Such doubtful accuracy may arise in deep water, as well as in very shallow water because we cannot use so minute meshes as to divide the shallow water depth. We should examine bottom-friction models including empirical friction laws, such that we introduce some adequate friction model into STOC to obtain more reasonable solutions also in field-scale calculation.



(a) Time variation of water level at Channel 5



(b) Time variation of water level at Channel 7



(c) Time variation of water level at Channel 9

Figure 5. Calculation results of time variation of water level by STOC-IC and STOC-VF

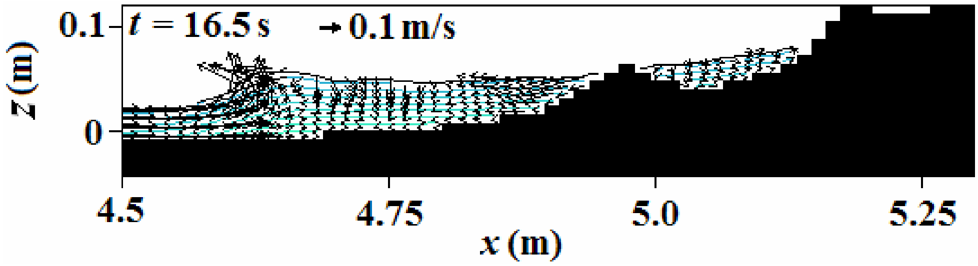


Figure 6. Calculation results by STOC-VF of velocity vectors and isobaric curves in the vertical section parallel to the axis *y* (In this section the tsunami shows the highest runup when *t* = 16.5 s.)

According to Figure 5, the 2D-calculation results by Yasuda & Watanabe (2005) based on the non-linear long-wave equations including friction terms with relative roughness show some time-lag of peak-value appearance in comparison with the experimental data. This tendency is not so strong as but similar to the results calculated by STOC-IC under the non-slip-bed condition, where the peak values appear later than those of the experimental data. It should be noted that the calculation results obtained by both STOC-IC and STOC-VF under the slip-bed condition capture the accurate time of peak-value appearance in the experiment successfully.

3) Runup due to the Okushiri tsunami calculated by STOC-VF under the slip-bed condition

In this subsection we assume that the sea bottom is a slip bed. We can treat runup of waves using STOC-IC or STOC-VF. In this paper we show calculation results of runup obtained by STOC-VF.

Figure 6 shows the results, which were calculated by STOC-VF, of velocity vectors and isobaric curves in the vertical section parallel to the axis *y* at the time when *t* = 16.5 s. In this figure the tsunami shows the highest runup. We can evaluate 3D flow considering effect of not only hydrostatic but also dynamic pressure using STOC-VF as well as STOC-IC. In order to know how dynamic pressure affects tsunamis over a complex topography, it is worthwhile, for example, to compare calculation results by such non-hydrostatic models as STOC-IC and STOC-VF, which solve Poisson equation of pressure, with those by hydrostatic models including nonlinear long-wave models. Precise evaluation of pressure is important especially when we study interaction among tsunamis, structures, driftage and ground, which requires future work.

Figure 7 shows wave profiles of the Okushiri tsunami calculated by STOC-VF. After draw-down, see Figure 7(a), the tsunami travels towards onshore. Around Point A, diffracted waves meet together behind Muen Island. We can see the valley or gulley, where the tsunami is going to show the maximum height of runup, in the top center of Figure 7(a).

In Figure 7(b) the tsunami reaches the land.

In Figure 7(c) we can see reflected waves from the coasts.

At Point B in Figure 7(d), a breaking wave is generated by superposition of waves, which have come from two different coasts, showing the largest value of wave height through the whole computation period except over the land area.

The maximum runup appears at Point C in Figure 7(e). This runup height is about 7.65 cm in the model scale, according to altitude of the intersections of water surface in consideration of the VOF function and vertical faces of meshes including porosity to describe the configuration of seabed smoothly. Thus the result says that the tsunami got the runup height of about 30.6 m in the prototype scale. This value is closer to the discovered value in the field, i.e., about 32 m, than the calculation results by Yasuda & Watanabe (2005), who utilized the 2D nonlinear long-wave models with the same size of horizontal meshes as the present calculation, resulting in about 28.0 m of runup height using square meshes and about 29.0 m using triangular improved meshes.

It is animations, which are prepared by joining figures, e.g. Figures 7(a) – 7(f), that are expected to deepen our understanding of tsunami phenomena owing to visual and dynamic presentation of numerical calculation results, as well as to be helpful for residents to take refuge quickly and adequately.

4 CONCLUSIONS

Two 3D numerical models built in STOC, i.e., STOC-IC and STOC-VF, have been applied to sea-water motion due to the 1993 Okushiri tsunami.

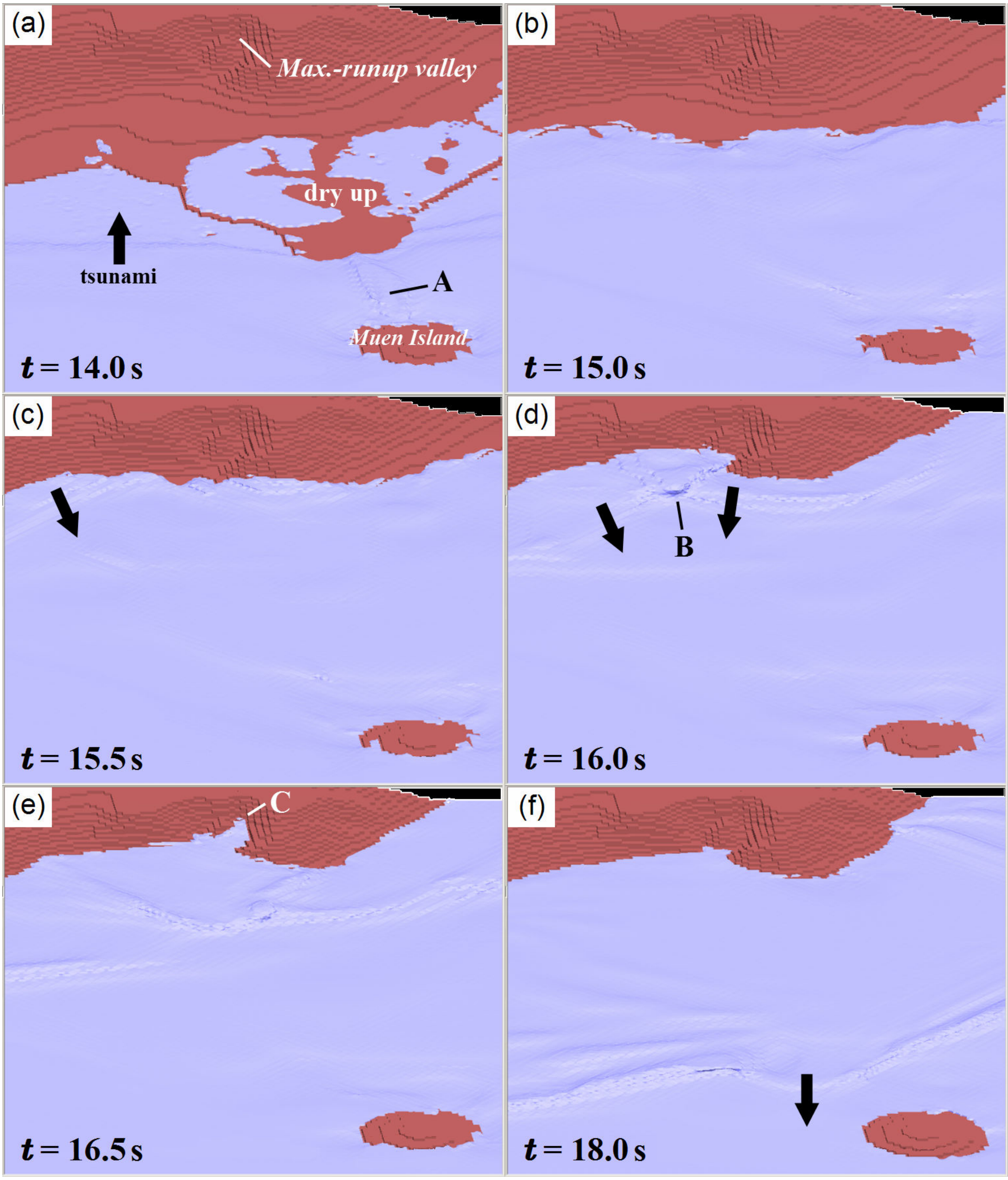


Figure 7. Runup due to the Okushiri tsunami calculated by STOC-VF

Their governing equations are the continuity and Reynolds-averaged Navier-Stokes equations for incompressible-fluid motion in porous media. Water surface displacement is determined by the vertically integrated equation of continuity in STOC-IC, while by the 3D-VOF method in STOC-VF. In numerical computation with these two models, we need neither long-wave assumption, irrotational assumption nor hydrostatic assumption, such that we can take into account vertical component of velocity including vorticity, as well as dynamic pressure. The calculation results by STOC-IC and STOC-VF are compared with both the existing experimental data by Matsuyama & Tanaka (2001) and the 2D-calculation results by Yasuda & Watanabe (2005) with the nonlinear long-wave models, leading to thefollowing results:

1) The results of water level calculated by STOC-IC and STOC-VF under the slip-bed condition are similar when there exists no breaking wave.

2) These two models generally give correspondent results of water level with the experimental data at off-peak times of the water level. In comparison with the experimental data, the results of water level calculated by STOC-IC and STOC-VF show higher first-peak values on the slip seabed.

3) The results calculated by STOC-IC under the non-slip-bed condition show too lower water level than that observed in the experiment. Some bottom-friction model is required.

4) The calculation results of water level obtained by both STOC-IC and STOC-VF under the slip-bed condition capture the accurate time of peak-value appearance in the experiment successfully, while the peak values calculated by both STOC-IC under the non-slip-bed condition and the 2D nonlinear long-wave model with relative roughness of the seabed appear later than those of the experimental data.

5) The highest runup is about 7.65 cm, which corresponds to 30.6 m in the prototype scale. This value, which was obtained by STOC-VF, is larger than that evaluated by use of the 2D nonlinear long-wave model adopting the same size of numerical meshes.

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